RELATIONSHIPS BETWEEN TROPICAL PRECIPITATION AND KINEMATIC CLOUD MODELS

REPORT NO. 3

Contract No. DA - 36-039 SC 89099

DA Project No. 3A 99-27-005

Third Quarterly Progress Report

1 November 1962 - 31 January 1963

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OBJECT

The object of this research is to study relationships between tropical precipitation and the associated circulations of water vapor and condensate.

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PURPOSE

The purpose of the project is to increase understanding of the roles of cloud conversion, accretion, evaporation, and entrainment processes in shaping the distributions of water vapor, cloud, and precipitation associated with tropical circulations.

ABSTRACT

A set of equations describing kinematical relationships among cloud, precipitation, and the wind field are derived in detail. Steady-state profiles of cloud in a non-precipitating tropical atmosphere are presented for 10°C steps of the condensation level; the unresponsiveness of cloud content to vertical air displacements at high altitudes and low temperatures is an explanation for the relative persistence of cirrus clouds. Exact solutions are presented for a cloud and precipitation model that is simplified to include only the problem of precipitation initiation. This model shows that where two processes for precipitation formation exist, the onset of precipitation is controlled only within certain limits by alteration of one of the processes. Microphysical parameters influence transient features of the development of precipitation and cloud. In many cases, however, the steady-state surface precipitation rate is only slightly responsive to order-of-magnitude changes of the microphysical parameters.

PUBLICATIONS, LECTURES, REPORTS, AND CONFERENCES

The paper "Elementary theory of associations between atmospheric motions and distributions of water content" by E. Kessler was published in the January, 1963 issue of the Monthly Weather Review.

The paper "Relationships between tropical precipitation and kinematic cloud models" by E. Kessler was presented at the 43rd Annual Meeting of the American Meteorological Society in New York City on 21 January 1963.

Dr. Kessler and Mr. Feteris met with Dr. H. Weickmann at the New York meetings and discussed the work of the contract and future plans.

The program committee of the 10th Weather Radar Conference has accepted our paper for presentation on April 23, 1963. The abstract follows:

Role of Microphysical Processes in Shaping
Vertical Profiles of Precipitation and Cloud

by

Edwin Kessler, Pieter J. Feteris, Edward A. Newburg

Continuity equations are presented which relate cloud and precipitation development to the wind field and processes of condensation, evaporation, conversion of cloud to precipitation, and collection of cloud by precipitation. The shape in steady updrafts of transient features of cloud and precipitation profiles is related to cloud conversion and collection rates, which control the onset of precipitation. However, if one of these processes is relatively rapid, the rate of development of precipitation is quite insensitive to the magnitude of the other. After precipitation starts, hydrometeor profiles approach a steady state in all cases where the maximum steady updraft is less than typical precipitation fall speeds. The equations constitute a model precipitation system whose reaction to changes of the microphysical processes is usually in a direction to minimize the effects of the changes. The steady-state vertical profiles are independent of initial conditions, weakly dependent on the microphysical parameters and strongly dependent on the updrafts.

1.0 FACTUAL DATA

1.1 Comprehensive Derivation of the Basic Equations

A detailed derivation of the continuity equations which are the foundation of this study has not been given previously. This derivation is of particular importance to two-dimensional problems now being designed and programmed for solution by digital computer.

1.1.1 A continuity equation for precipitation

Assume that the fall speed of the precipitation content M relative to the air at any one height and time can be represented by the parameter V; if the precipitation content at a point is distributed over particles of different sizes and fall speeds, V at that point must be an average value. Then a development practically identical to that in Haurwitz [4, p. 128], for example, gives the following fundamental continuity equation for precipitation content M,

$$\frac{\partial M}{\partial t} = -\left[\frac{\partial}{\partial x} Mu + \frac{\partial}{\partial y} Mv + \frac{\partial}{\partial z} M(w + V)\right]. \qquad (1)$$

The horizontal winds u and v are in the x- and y- directions, respectively, and the vertical air speed is w. This equation governs the distribution of M already formed but makes no allowance for the creation of M (creation of M is considered in section 1.1.3 below).

Assume that horizontal and local time changes of air density ρ are relatively small. Then the air motions are in close accord with the equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -w\frac{\partial \ln \rho}{\partial z} \qquad (2)$$

Substitution of Eq. (2) into Eq. (1) yields

$$\frac{\partial M}{\partial t} = -u\frac{\partial M}{\partial x} - v\frac{\partial M}{\partial y} - (w+V)\frac{\partial M}{\partial z} - M\left[\frac{\partial V}{\partial z} - w\frac{\partial \ln \rho}{\partial z}\right] \qquad (3)$$

The derivatives of the wind appearing in Eq. (1) are absent from Eq. (3); the density term which replaces them accounts for the compressibility of the atmosphere.

1.1.2 A continuity equation for cloud and vapor

A tendency to supersaturation in the atmosphere is rapidly offset by condensation of tiny cloud particles; cloud particles in clean air evaporate in a matter of seconds when the relative humidity is more than a few percent below saturation [5, 7]. These properties of cloud suggest our definition of m: m is the cloud density minus the saturation vapor density plus the actual vapor density, i.e.,

$$m = \text{cloud density} + Q - Q_s = \mathcal{M} - Q_s, \qquad (4)$$

where Q is the actual vapor density, Q_s is the saturation vapor density, and $\mathcal{M}$ is the total content of water substance excepting precipitation.

A continuity equation for m is derived by first considering that for $\mathcal{M}$, which is similar to Eq. (3), except for the absence of terms like $M\frac{\partial V}{\partial z}$ and $V\frac{\partial M}{\partial z}$:

$$\frac{\partial \mathcal{M}}{\partial t} = -u\frac{\partial \mathcal{M}}{\partial x} - v\frac{\partial \mathcal{M}}{\partial y} - w\frac{\partial \mathcal{M}}{\partial z} + w\frac{\partial \ln \rho}{\partial z} \qquad (5)$$

Substitute for $\mathcal{M}$ in (5) from (4) to obtain

$$\frac{\partial(m + Q_s)}{\partial t} = - u \frac{\partial(m + Q_s)}{\partial x} - v \frac{\partial(m + Q_s)}{\partial y} - w \frac{\partial(m + Q_s)}{\partial z} + (m + Q_s)\, w \frac{\partial \ln \rho}{\partial z} \tag{6}$$

Assume that the saturation vapor density of the air is locally steady and horizontally uniform, i.e., that $\frac{\partial Q_s}{\partial t} = \frac{\partial Q_s}{\partial x} = \frac{\partial Q_s}{\partial y} = 0$. Even though this is rarely exactly true, it is usual for the local time changes and horizontal variations of Q_s to be quite small compared with the vertical variations [cf. assumptions implicit in Eq. (2)]. Then Eq. (6) becomes

$$\frac{\partial m}{\partial t} = - u \frac{\partial m}{\partial x} - v \frac{\partial m}{\partial y} - w \frac{\partial m}{\partial z} + mw \frac{\partial \ln \rho}{\partial z} - w\left(\frac{\partial Q_s}{\partial z} - Q_s \frac{\partial \ln \rho}{\partial z}\right) \tag{7}$$

Equation (7) is significantly unlike Eq. (3). Since m has no fall speed relative to the air, Eq. (7) does not contain terms in the fall speed of m; also, since the saturation deficit or cloud content changes when the air is displaced vertically, Eq. (7) contains terms in the saturation density Q_s which account for these changes. The sum of the terms in Q_s in (7) we call the generating function G; a fair approximation to G in the real troposphere is a linear function of height.

1.1.3 A system of continuity equations for vapor, cloud, and precipitation including cloud-precipitation interactions

The modification of Eqs (3) and (7) to model the bulk effects of interactions among the three-dimensional cloud and precipitation particles as well as their distribution by wind, is based on elementary considerations of precipitation physics. The first product of rising motion is cloud. Coalescence of cloud particles to form precipitation is apparently favored by the presence of a broad spectrum of cloud particle sizes, e.g., by the presence of large hygroscopic salt nuclei in the subsaturated atmosphere [6]. Growth of tiny ice particles to precipitation size may also occur by deposition when liquid and ice phases coexist at subfreezing temperatures [6, 7]. Either of these processes may be represented by a term of appropriate magnitude called "autoconversion of cloud."

Once precipitation particles are formed, their fall speed carries them through the population of cloud particles and facilitates the rapid growth of precipitation by collection of cloud [8]. This process is represented by a term whose magnitude must increase with the content of cloud and precipitation; this term called "collection of cloud", contributes equally to development of precipitation and depletion of cloud.

The third processes of particular interest is evaporation. The terms in Q_s in Eq. (7) require that evaporation of cloud in downdrafts is just rapid enough to maintain saturation. The evaporation of precipitation in unsaturated (cloud free) air requires an additional term.

Equations (3) and (7) are now rewritten with the addition of terms representing the transfer of precipitation, cloud, and vapor from one phase to another.

$$\frac{\partial M}{\partial t} = -u\frac{\partial M}{\partial x} - v\frac{\partial M}{\partial y} - (w+V)\frac{\partial M}{\partial z} - M\left[\frac{\partial V}{\partial z} - w\frac{\partial \ln\rho}{\partial z}\right] \quad (8)$$

+ autocon. of cloud + collection of cld. - evap. of precip.

$$\frac{\partial m}{\partial t} = -u\frac{\partial m}{\partial x} - v\frac{\partial m}{\partial y} - w\frac{\partial m}{\partial z} + mw\frac{\partial \ln\rho}{\partial z} + wG \tag{9}$$

$$- \text{autocon. of cloud} - \text{collection of cld.} + \text{evap. of precip.}$$

In Eq. (9), $G = Q_s\frac{\partial \ln\rho}{\partial z} - \frac{\partial Q_s}{\partial z} = -\rho\frac{\partial Q_s'}{\partial z}$, where Q_s is the saturation vapor density, and Q_s' is the saturation mixing ratio. The autoconversion and collection terms in (8) and (9) are non-zero only when $m > 0$, i.e., when there is cloud. As noted above, the evaporation term is non-zero only when $m < 0$, i.e., when the air is not saturated with vapor.

Equations (8) and (9) can be reformulated to refer to mixing ratio units by substituting $\rho m'$ wherever m appears, $\rho M'$ wherever M appears, and $\rho Q'$ wherever Q appears. After simplification, there is obtained:

$$\frac{\partial M'}{\partial t} = -u\frac{\partial M'}{\partial x} - v\frac{\partial M'}{\partial y} - (w+V)\frac{\partial M'}{\partial z} - M'\left(\frac{\partial V}{\partial z} + V\frac{\partial \ln\rho}{\partial z}\right) \tag{8a}$$

$$+ \text{autocon. of cld.} + \text{cld. collec.} - \text{evap. of precip.}$$

$$\frac{\partial m'}{\partial t} = -u\frac{\partial m'}{\partial x} - v\frac{\partial m'}{\partial y} - w\frac{\partial m'}{\partial z} - w\frac{\partial Q_s'}{\partial z} \tag{9a}$$

$$- \text{autocon. of cld.} - \text{cld. collec.} + \text{evap. of precip.}$$

where the primed quantities are in mixing ratio units.

The absence of the density term in (9a) is due to the conservative nature, following the motion of air particles, of the mixing ratio of water vapor plus cloud. Although (9a) is therefore slightly simpler than Eq. (9), our studies use Eqs. (8) and (9) in density units because radar reflectivity characteristics, visual appearance of clouds, and certain physical effects are more easily understood in these units.

1.2 Relation of Present Model to Previous Work

The published works [2, 3, 4] which form a principal foundation to the studies of this contract, treat a cloud-free model atmosphere in which condensate appears instantly as precipitation of fall speed V. Equations (8) and (9) can be reduced to the basic equation of the earlier model. Thus, the addition of Eqs. (8) and (9) yields

$$\frac{\partial(M+m)}{\partial t} = -u\frac{\partial(m+M)}{\partial x} - v\frac{\partial(m+M)}{\partial y} - w\frac{\partial(m+M)}{\partial z} - \frac{\partial}{\partial z}MV + (m+M)\,w\frac{\partial \ln \rho}{\partial z} + wG \; . \tag{10}$$

If $M + m$ is denoted by a new variable M^* and if it is assumed that $m = 0$ when $M^* > 0$ and that M and V are zero when $M^* < 0$, Eq. (1) becomes

$$\frac{\partial M^*}{\partial t} = -u\frac{\partial M^*}{\partial x} - v\frac{\partial M^*}{\partial y} - w\frac{\partial M^*}{\partial z} - \frac{\partial}{\partial z}M^*V + M^*w\frac{\partial \ln \rho}{\partial z} + wG \tag{11}$$

$$\begin{cases} m = 0 \text{ when } M^* > 0 \\ M = V = 0 \text{ when } M^* < 0 \end{cases}$$

This is the three-dimensional formulation of the model system that has been treated extensively in the earlier work. M^* when negative is the saturation deficit which follows the motion of the air; when positive, M^* is the precipitation content which falls at speed V relative to the air. Equation (11) models a system wherein cloud does not exist because condensate appears instantly as precipitation.

1.3 Steady-state Vertical Distribution of Cloud in a Precipitation-free Compressible Atmosphere

Section 1.3 in Report No. 2 discusses the steady-state vertical profile of cloud in a precipitation-free compressible atmosphere as modeled by the equation

$$\frac{\partial m}{\partial z} = m \frac{\partial \ln \rho}{\partial z} + G . \qquad (12)$$

The <u>compensation level</u> is defined as that height where $m \frac{\partial \ln \rho}{\partial z} = -G$, i.e., where the tendency of the generating function to increase the density of condensate is exactly compensated by the decreasing density of ascending air. Above the compensation level, cloud density decreases with ascent and increases with descent. In Report No. 2, a function linear with height is assumed for G, and the corresponding solutions of Eq. (12) are presented in Fig. 4 of that report. After Report No. 2 was written, a more accurate profile of the steady-state cloud content was derived. Since no relatively simple formula has been found to approximate the natural generating function at high levels (low temperatures) with the desired accuracy, the revised cloud profiles have been computed with the aid of the Smithsonian Meteorological Tables [9, Tables 71, 78, and 108].

The results for an atmosphere of constant wet-bulb potential temperature 295° are shown in Fig. 1. Note that in the upper troposphere, (ice) cloud amounts less than 0.1 gm m^{-3} may be associated with anomalous behaviour, i.e., decreasing cloud density with ascending motion, and conversely. Accurate measurements of water content and temperature in dense tropical cirrus would be of special interest in connection with the application of this theory to problems concerning the development and persistance of high cloud.

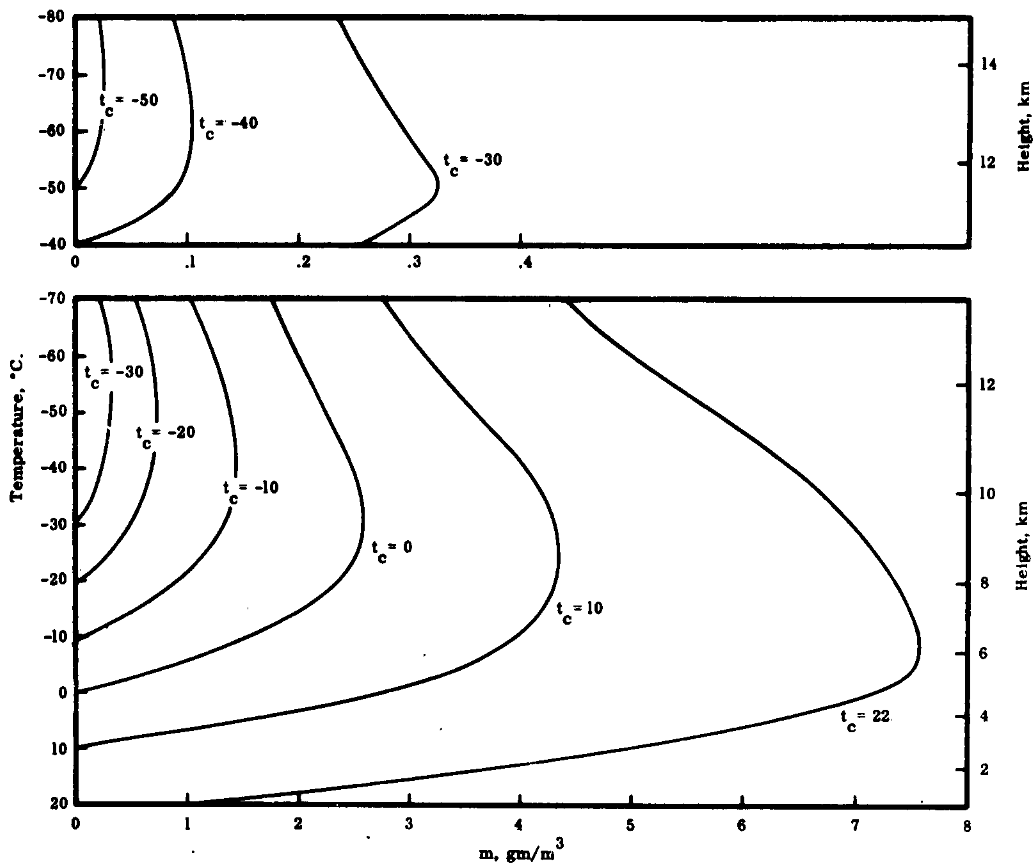

Fig. 1 Steady-state cloud profiles in a non-precipitating atmosphere with wet bulb potential temperature = 22°C.

The data represented by Fig. 1 vary only slowly with height for constant temperature; this figure can therefore be applied to various elevations where the same temperatures occur in temperate and arctic regions.

1.4 A Model of the Onset of Precipitation

The cloud conversion and accretion terms in (8) and (9) have been discussed in the previous reports on this contract. The natural processes are modeled as follows:

$$\begin{aligned} &\text{Cloud autoconversion} = k_1(m - a) && (k_1 = 0 \text{ when } m < a) \\ &\text{Accretion} = k_2 E n_0^{.125}\, mM^{.875} && (k_2 = 0 \text{ when } m < 0) \end{aligned} \qquad (13)$$

In (13), k_1 and a are parameters whose magnitudes are selected to model spontaneous conversion of cloud at rates in accordance with theories and observations, $k_2 = 7 \times 10^{-4}$, n_0 defines the number of small drops in an assumed Marshall-Palmer distribution of precipitation particles, and E is the efficiency with which precipitation particles collect cloud. Units are gms, meters, and seconds throughout.

In section 1.5 of Report No. 2 the onset time of precipitation in the incompressible one-dimensional model version of Eqs. (8) and (9) is related to the magnitude of k_1. For values of the other parameters which are considered to be reasonable approximations to natural values, the onset time of precipitation at the ground was found to increase only about 10 minutes with a change of k_1 from 10^{-1} sec^{-1} to 10^{-5} sec^{-1}. In order to understand better how the cloud conversion and accretion coefficients influence the onset of precipitation, exact solutions of a simple model which embraces only the initiation problem are studied.

Consider the terms (13) as they operate in a deep model cloud of uniform water content without vertical air motion. In this model, cloud changes to precipitation everywhere at the same rate, the advection terms vanish, and the equations describing the amounts of cloud and precipitation are

$$\frac{dM}{dt} = -\frac{dm}{dt} = k_1(m-a) + k_2En_0^{.125}\,mM^{.875} \qquad (14)$$

$$k_1 = 0 \text{ when } m < a;$$
$$k_1 > 0 \text{ when } m > a;$$
$$k_2 > 0 \text{ always .}$$

Since there is no vertical motion and therefore no condensation, the condition

$$M + m = m(t=0) = m_0, \quad 0 \leq t \leq \infty \qquad (15)$$

where m_0 is the initial cloud amount, must also be satisfied. If the exponent of M in (14) is changed from $+0.875$ to $+1.0$, exact integration is grately facilitated without changing essential properties of the formulation. Then substitution of (15) into (14) with altered exponent yields

$$\frac{dM}{dt} = -\frac{dm}{dt} = k_1(m-a) + C_2m(m_0-m), \qquad (16)$$

where $C_2 = k_2\,En_0^{.125}$.

The interval t_a required for the cloud amount to decrease from the amount m_0 to the amount a, when the autoconversion term ceases to be a factor, is a convenient measure of the time of onset of precipitation:

$$t_a = \int_{m_0}^{a} \frac{dm}{C_2m^2 - (k_1 + C_2m_0)\, m + k_1a} \tag{17}$$

The integral of the right hand side depends on the size of the discriminate $\mathcal{D}$ of the denominator

$$\mathcal{D} = k_1^{\,2} + 2k_1C_2m_0 + C_2^{\,2}m_0^{\,2} - 4k_1C_2a \tag{18}$$

That $\mathcal{D} > 0$ can be shown in the following way. Note first that k_1, C_2, m_0, and a are all positive. Therefore, the first three terms on the right hand side are positive. Set $m_0 = a$; if $\mathcal{D} > 0$ when $m_0 = a$, the same is certainly true when $m_0 > a$. Thus

$$\mathcal{D} > k_1^{\,2} + 2k_1C_2a + C_2a^2 - 4k_1C_2a \tag{19}$$

and

$$\mathcal{D} > (k_1 - C_2a)^2 \tag{20}$$

Obviously the minimum value of the expression on the right hand side is zero and the discriminant is therefore positive. Then

$$t_a = \left[(k_1 + C_2m_0)^2 - 4k_1C_2a\right]^{-\frac{1}{2}} \times \ln \frac{2C_2m - (k_1 + C_2m_0) - \left[(k_1 + C_2m_0)^2 - 4k_1C_2a\right]^{\frac{1}{2}}}{2C_2m - (k_1 + C_2m_0) + \left[(k_1 + C_2m_0)^2 - 4k_1C_2a\right]^{\frac{1}{2}}} \Bigg|_{m_0}^{a} . \tag{21}$$

When $m_0 = 2a$, this can be written

$$t_a = \frac{1}{s} \ln \, (S)(\mathcal{J}) \tag{22}$$

where

$$s = (k_1 + 4C_2{}^2 a^2)^{\frac{1}{2}} ,$$

$$S = \left[\frac{-1 - \left(1 + \dfrac{4C_2{}^2 a^2}{k_1{}^2}\right)^{\frac{1}{2}}}{-1 + \left(1 + \dfrac{4C_2{}^2 a^2}{k_1{}^2}\right)^{\frac{1}{2}}} \right] \tag{23}$$

$$\mathcal{S} = \left[\frac{2a\dfrac{C_2}{k_1} - 1 + \left(1 + \dfrac{4C_2{}^2}{k_1{}^2} a^2\right)^{\frac{1}{2}}}{2a\dfrac{C_2}{k_1} - 1 - \left(1 + \dfrac{4C_2{}^2}{k_1{}^2} a^2\right)^{\frac{1}{2}}} \right] .$$

Note that S and $\mathcal{S}$ for specified a depend only on the ratio C_2a/k_1 ; all the terms are dependent on k_1 alone when the term C_2a is fixed.

The variation of t_a with C_2a for $m_0 = 2a$ and for various values of the cloud conversion coefficient k_1 is shown in Fig. 2. Note that when $k_1 >> C_2a$, the time t_a is a very slowly varying function of C_2a. And when $k_1 << C_2a$, t_a is a very slowly varying function of k_1 . In other words, if the coalescence and aggregation processes among the many tiny cloud drops are relatively rapid, the efficiency with which the larger raindrops collect cloud is of little consequence to the rate of depletion of cloud; and if the collection of cloud by the raindrops is relatively rapid, then the magnitude of the rate of cloud conversion is not critical, so long as it is more than zero.

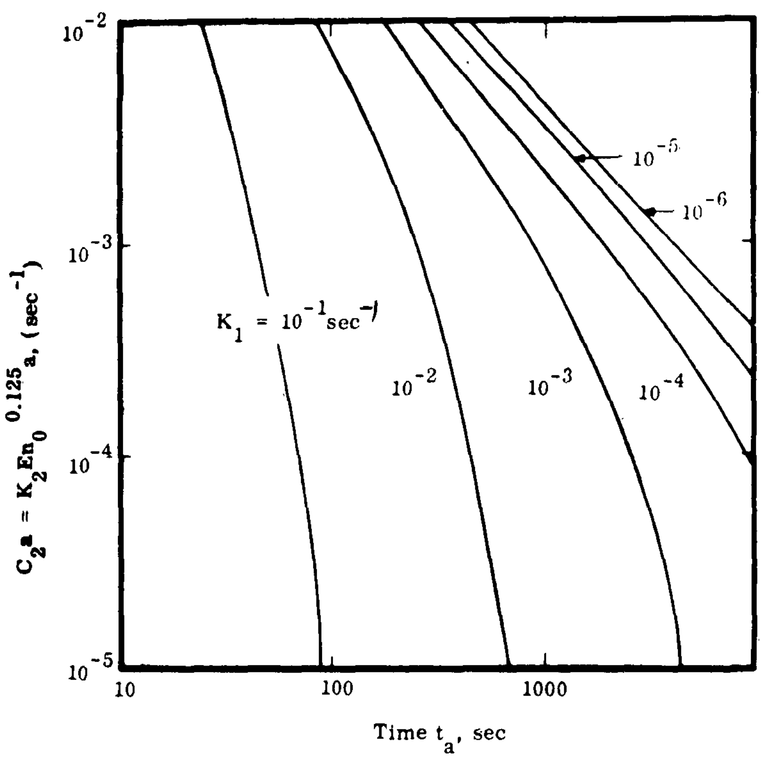

Fig. 2 Variation of the onset parameter t_a with the conversion parameter k_1 and with accretion. It is assumed that $m_0 = 2a$.

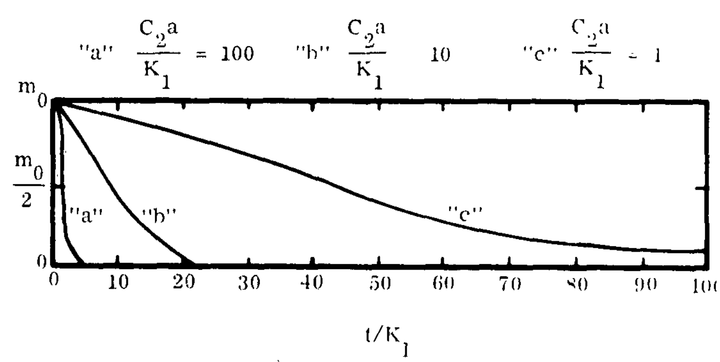

Fig. 3 Detailed development with time of precipitation in the model defined by Eq. (16) with $m_0 = 2a$.

In Fig. 2, the region $10^{-3} < C_2a < 10^{-2}$ holds the greatest interest because it is here that most empirical data are represented. For example, the heavy horizontal line indicates the following parameter combination:

$$a = 1 \text{ gm m}^{-3},\ m_0 = 2 \text{ gm m}^{-3},\ E = 1, \text{ and } n_0 = 10^7 \text{ m}^{-4} .$$

If $a = 0.5$ gm m^{-3}, $m_0 = 1$ gm m^{-2}, $E = .5$, and $n_0 = 10^7$ m^{-4} then $C_2a = 1.25 \times 10^{-3}$. The values of k_1 in the range $10^{-6} \leq k_1 \leq 10^{-1}$ sec^{-1} for the range $10^{-3} \leq C_2a \leq 10^{-2}$ are associated with onset parameters t_a ranging from about one minute to about on hour.

When $m \leq a$, $k_1 = 0$ and the solution of (16) is simply

$$m = \frac{2a}{1 + \exp 2C_2a(t - t_a)} . \tag{24}$$

This equation has contributed to the derivation of the curves in Fig. 3 which illustrate in detail the time dependence of the cloud content for three values of C_2a/k_1. Curves of the precipitation content M mirror those shown; they start at $M = 0$ and asymptotically approach the value m_0 as the time increases.

1.5 Relationships Between Microphysical Parameters and Transient Features of Solutions of the Complete Model

The one-dimensional forms of Eqs. (8) and (9) for an incompressible atmosphere have been solved by an IBM 7090, programmed to use the finite-difference equations discussed in Report No. 1. In each case discussed here, the initial condition is assumed to be a saturated atmosphere without cloud or precipitation. The updraft $w = (\frac{4w_{max}}{H})$

$(z - z^2/H)$, where $w_{max} = 0.5$ m/sec and H , the top of the updraft column, is 6 km in all cases. The generating function $G = 3 \times 10^{-3} - 3 \times 10^{-7} z$, is a fair approximation to the lower troposphere in the tropics. This section discusses the role of the model microphysical parameters in shaping transient features of the solutions.

1.5.1 The onset parameter a

The model provides that conversion of cloud to precipitation does not start until the cloud content produced in saturated updrafts has exceeded the magnitude a . Thereafter, cloud responds to its accretion by relatively large precipitation particles, in addition to advection and condensation processes. Often, the amount of cloud which can coexist in equilibrium with precipitation is less than the amount required to initiate precipitation in the model. When this is the case, the time dependent solutions for cloud and precipitation are characterized by an initial pulse whose height is about proportional to the difference between the equilibrium precipitation content and the magnitude of a .

A mathematical approach to this problem can be illustrated for a place on the cloud profile where the cloud content is at or near its maximum and the advection of cloud is therefore small, and where the steady-state precipitation rate is nearly equal to the condensation rate integrated over all higher elevations.* Then

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* i.e., the equilibrium cloud content is small compared to its steady-state value in the absence of precipitation, and updrafts are small compared to precipitation fall speeds (cf, Report No. 2, Section 1.2).

$$R = -M(V+w) \approx \int_z^H wGdz \,, \tag{25}$$

where R is the precipitation rate at height z ; V , the fall velocity of M , discussed in the earlier reports, is $V = -38.6\ n_0^{-.125} M^{.125}$, and H is the top of the saturated updraft column. An equilibrium between condensation and accretion processes is represented by the equation

$$0 = wG - k_2\, En_0^{.125}\, mM \,. \tag{26}$$

By carrying out the integration in (25) for a parabolic w- profile and the generating function $G = A + Bz$, and making other obvious substitutions, Eqs. (25) and (26) can be combined to obtain the following expression of the maximum cloud amount m_{ep} which can coexist in equilibrium with precipitation:

$$m_{ep} = \frac{3.35 \times 10^4}{E} \left(\frac{w_{max}}{n_0 H^2}\right)^{2/9} \left[\Phi\right]\left[\Psi^{-7/9}\right]. \tag{27}$$

In Eq. (27), Φ and Ψ are,

$$\begin{aligned} \Phi &= AHz - (A - BH)\,z^2 - Bz^3 \\ \Psi &= \left[AH(H^2 - z^2)/2\right] - \left[(A - HB)(H^3 - z^3)/3\right] - \left[B(H^4 - z^4)/4\right]. \end{aligned} \tag{28}$$

It is interesting that greatly simplifying assumptions still lead to quite complicated expressions. The magnitude of the initial pulse of cloud, which is manifested in the time variations of precipitation at the surface, can be estimated by comparing the magnitude of m given by (27) with the magnitude of a . Plots of Eq. (27) will be presented in the next report.

1.5.2 The cloud conversion parameter k_1

In the model, the parameter k_1 is the rate of autoconversion to precipitation of cloud content in excess of the magnitude a. When k_1 is large, the approach to a steady-state condition is expedited after m exceeds a; when k_1 is small, the cloud content continues to increase in the updrafts after precipitation starts (especially if C_2a is also small). Thus the height of a pulse-shaped transient near the start of precipitation is enhanced as the value of k_1 decreases. This is illustrated for three values of k_1 in Fig. 4a. Nearly the same steady state is ultimately approached in all three cases because the cloud content in equilibrium with precipitation for the indicated choice of parameters is less than the amount a at most levels, and the magnitude of k_1 is therefore rather unimportant after the steady-state condition in these cases has been established. (See Fig. 5b, section 1.6.2.)

1.5.3 The collection efficiency E

The model permits study of the role in the precipitation process of the efficiency with which precipitation collects cloud. Figure 4b shows the development of precipitation at the ground in a case where the collection efficiency E is unity and in a case where $E = 0$. The steady-state precipitation rate is lower in the latter case, because more condensate remains as cloud aloft and is spread horizontally by high level divergence of the wind. When E is greater than zero, two processes can contribute to precipitation formation; when E is relatively very small or zero, the autoconversion process is the only important one.

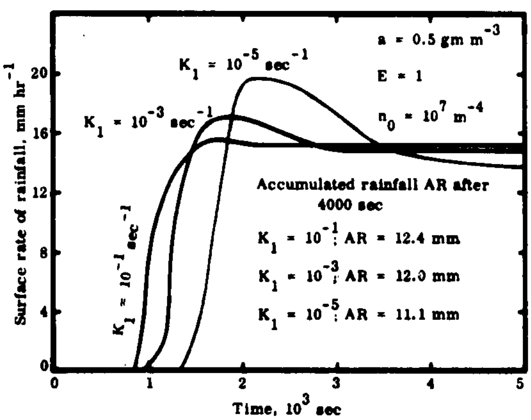

Fig. 4a Development of precipitation at the ground for various magnitudes of the conversion parameter k_1.

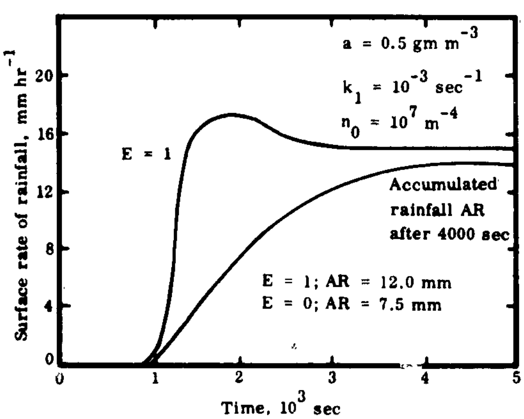

Fig. 4b Development of precipitation at the ground for extrema of the collection efficiency E.

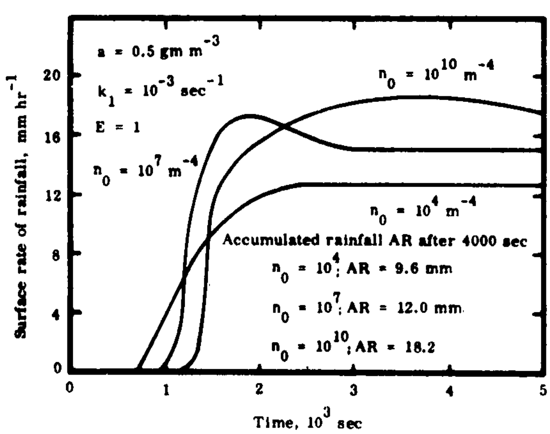

Fig. 4c Development of precipitation at the ground for three values of n_0. The value $n_0 = 10^7 m^{-4}$ is typical of natural rains.

1.5.4 The precipitation drop size parameter n_0

For at least two reasons, the parameter n_0 affects the cloud and precipitation distributions. First, the rate of cloud collection for fixed M and E increases slowly as n_0 increases. Second, as n_0 increases, the number of small drops increases, and therefore the average terminal velocity of drops decreases. However, both the collection and fall speed equations involve the eighth root of n_0, and order of magnitude changes of this distribution shape parameter are necessary to alter cloud and precipitation distributions significantly.

Figure 4c shows the time variation of precipitation at the ground accompanying three values of n_0. The middle curve is associated with an n_0 typical of natural rain. When n_0 is very small, particles fall rapidly and precipitation starts earlier at the ground. When n_0 is as large as $10^{10}\ m^{-4}$, however, the average particles are only the size of drizzle; their very slow falling speed is associated with both the delayed arrival of steady conditions, and the relatively great rate of steady precipitation of this case. The latter is associated with the air's horizontal convergence, implicit in the equations, which produces a progressively greater imbalance between condensation and precipitation rates at updraft centers as the ratio of fall speeds to updrafts approaches unity. As discussed previously (section 1.2 in Report No. 2) an excess or deficit of precipitation at updraft centers as compared to condensation there, is compensated at locations away from the center.

1.6 Relationships Between Microphysical Parameters and Steady-state Profiles of Cloud and Precipitation

This section discusses the role of the microphysical parameters in shaping steady-state solutions of the differential equations. The initial conditions and the parameters w_{max} , H and G are the same as noted in section 1.5. In section 1.5 the discussion of transients is illustrated by graphs of the development with time of the precipitation rate at the ground. In this section, the figures show steady-state vertical profiles of cloud and precipitation.

1.6.1 The onset parameter a

When the onset parameter a is quite large, the cloud amount is determined solely by a balance between condensation, vertical advection, and accretion processes. When a is small, however, the autoconversion and accretion processes both contribute to depletion of cloud, and the steady-state cloud content is therefore reduced.

The effect of varying a between particular values is shown in Fig. 5a. When a is large, the cloud content in the upper atmosphere tends to be large because it is here that precipitation is light and relatively ineffective in removing cloud. Some of the large amounts of cloud aloft are lost to precipitation at the updraft center by virtue of horizontal divergence of the wind [cf, Eq. (7) in Report No. 2], and the precipitation at the ground is therefore slightly reduced in this case.

Note that in both cases the cloud amount below 3.5 km is less than a and therefore locally regulated by the accretion, condensation, and advection.

1.6.2 The cloud conversion parameter k_1

Comparison of the steady-state profiles in Figs. 5a and 5b shows that the roles of k_1 and a are similar. Indeed, the cloud and precipitation profile in Figs. 5b for $k_1 = 10^{-5}$, $a = 0.5$ is virtually identical to the profile for $k_1 = 10^{-3}$, $a = 2.0$ in Fig. 5a. The similarity is not surprising, since autoconversion is made small either by increasing a or decreasing k_1 .

1.6.3 The collection efficiency E

The role of the collection efficiency E is illustrated by Fig. 5c.* When the collection efficiency is zero, the accretion process no longer contributes to depletion of cloud, and the cloud content rises. The curve shown for $E = 0$ applies to a balance between condensation, advection, and cloud autoconversion processes.

Note that the change of the cloud profile for the change of E from 1 to 0 is much greater than the accompanying change of the precipitation profile; this is true for the particular values of k_1 and w_{max} used because k_1 alone is large enough in this case to permit most of the cloud to convert to precipitation before being spread by horizontal divergence at high levels. In other words, the conversion time constant $1/k$ is much smaller ($1/k = 1000$ secs.) than the time that it takes an air parcel to ascend from low to high levels in the column ($H/w_{max} = 12{,}000$ secs.).

By setting $\frac{\partial m}{\partial t} = \frac{\partial m}{\partial z} = 0$ in the one-dimensional incompressible form of Eq. (9), one finds that when in the absence of accretion, the cloud profile has a maximum, that maximum satisfies the equation

* The curve for $E = 1$ in this figure also appears in Figs. 5a and 5b.

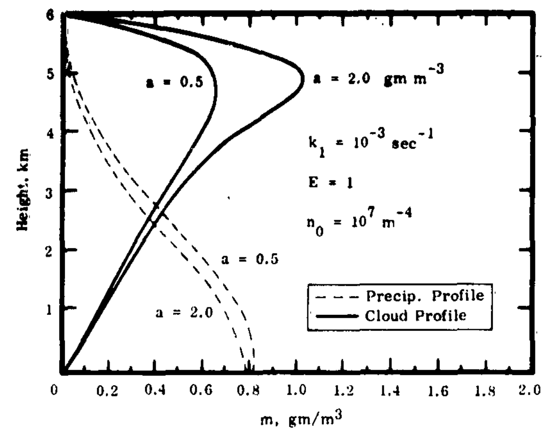

Fig. 5a Steady-state profiles of cloud and precipitation content for two values of the onset parameter a.

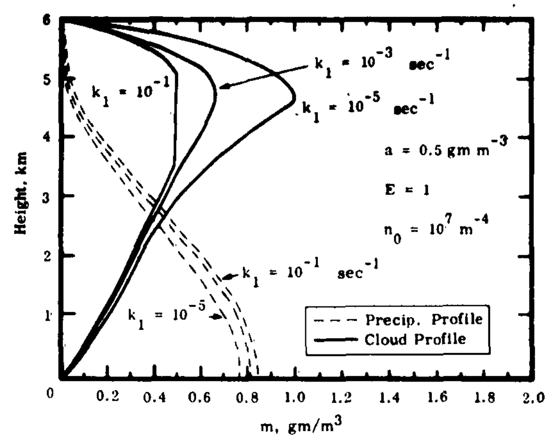

Fig. 5b Steady-state cloud and precipitation content profiles for three values of the cloud conversion parameter k_1.

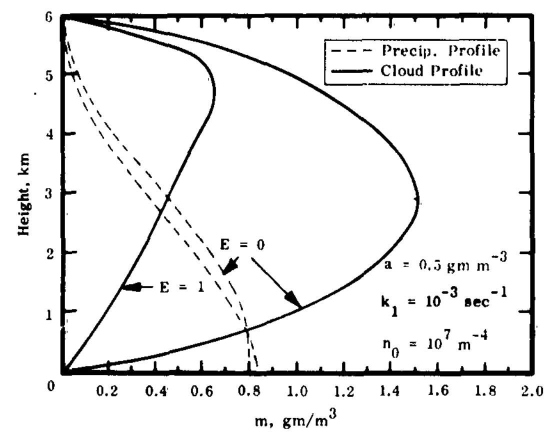

Fig. 5c Steady-state cloud and precipitation content profiles for extrema of the collection efficiency E.

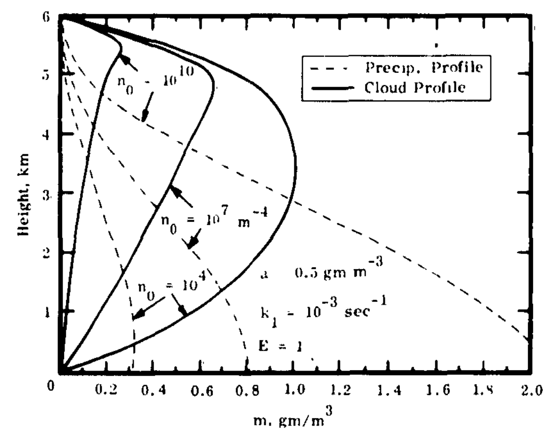

Fig. 5d Steady-state cloud and precipitation content profiles for three magnitudes of n_0.

$$m_{max} = \frac{wG}{k_1} + a \,. \tag{29}$$

If the location of m_{max} were fixed, its magnitude would rise indefinitely as k_1 decreases. Equation (29) is indeterminate, however, unless the location of m_{max} and the magnitude there of wG are ascertained. The maximum numerical value of m at any particular point can not be larger than the steady-state cloud content at that point in a nonprecipitating atmosphere. As k_1 becomes smaller, the m- profile in the absence of accretion more nearly resembles the steady-state profile defined by the equation

$$m = \int_0^z Gdz \;; \tag{30}$$

this equation has m increasing with z to the top of the updraft column.

When $1/k_1 >> \frac{w_{max}}{H}$, the cloud profile is accurately defined by Eq. (30)* and condensation is spread horizontally by high level divergence instead of producing rain at places beneath the updraft column.

1.6.4 The precipitation drop size parameter n_0

Much of the explanation of the curves shown in Fig. 5d is given in section 1.5.4. When n_0 is large, small slow falling drops are numerous and accretion of cloud is enhanced. Therefore, cloud amounts

---

* In the compressible case, the steady-state m- profiles in the absence of accretion and as k_1 approaches 0 , approach the curves discussed in section 1.3 of this report.

in this case are small. The precipitation content M is increased when n_0 is large and fall speeds small because there is then more time for the precipitation to develop from condensation in the updrafts. And the enhancement of the precipitation rate MV at updraft centers when fall speeds are small can be explained by the divergence term in Eq. (7), Report No. 1, whose contribution to the surface precipitation rate increases as the ratio of precipitation fall speeds to maximum updrafts approaches unity.

1.7 Computer Program for Two-dimensional Rectilinear and Radially Symmetric Models

Dr. Newburg is preparing specifications for a computer program to define cloud and precipitation distributions and budget parameters in two-dimensional rectilinear and radially symmetric model wind fields. An explicit, conditionally stable finite difference approximation to the partial differential equations will be used in this program.

1.8 References

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2.0 CONCLUSIONS

At heights greater than 12 km in the tropical atmosphere, cloud amounts smaller than .05 gm/m^3 are affected only slightly by vertical displacements of the order of a kilometer. This fact may explain relatively great persistence of cirrus cloudiness.

The cloud and precipitation model illustrates that when more than one process influences precipitation production, the onset of precipitation is controlled only within certain limits by alteration of one of the processes. Where two processes for precipitation formation exist, simultaneous changes in the same direction of the magnitude of both are required if the changes are to be linearly reflected in the time of onset and amount of precipitation.

The discussion of sections 1.5 and 1.6 show that the complete kinematical equations with specified updrafts is a model system whose reaction to changes of the microphysical processes is generally in a direction to minimize the effects of the changes on the surface precipitation rate. Thus, an increase of cloud collection efficiency is associated more with decreased cloud amounts than with increased precipitation. Although the fall speed of precipitation particles is closely associated with the distribution of precipitation in the neighborhood of an updraft center, the total amount of condensation and precipitation is most closely related to the strength, areal extent, and duration of updrafts.

The work should be of value for interpreting measurements of the development of cloud and precipitation distributions in terms of the accompanying wind fields and microphysical processes. It may also stimulate weather modification experiments designed to alter precipitation and cloud distributions.

The combination of thermohydrodynamic considerations with the kinematic theory would be an important achievement.

3.0 PROGRAM FOR THE NEXT INTERVAL

The following work and study has been planned for the next interval:

1) Study of the development of a model cloud-precipitation system from unsaturated initial conditions.

2) Examination of equations which relate the rate of evaporation of precipitation beneath a convective cloud to the saturation deficit, the rate of precipitation, and the vertical velocity profile.

3) Preparation of a computer program to derive cloud and precipitation distributions and budget parameters in two-dimensional rectilinear and radially symmetric model wind fields.

4) Preparation of the one-dimensional results for submission to the Journal of Atmospheric Sciences.

4.0 PARTICIPATION OF PERSONNEL

The persons listed in the following table contributed to the work of the contract during the past quarter.

Name	Title	Hours Worked (approximate)
E. Kessler	Principal Investigator	130
P.J. Feteris	Research Scientist(Meteorologist)	304
E.A. Newburg	Research Scientist(Mathematician)	100
G. Wickham	Research Associate(Programmer)	70

Secretarial, administrative, and drafting assistance was also provided.

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3rd Quart. Progress Rpt., 1 Nov. 1962–Jan. 31, 1963. 27 pp. incl. 5 figs., 7 refs. Contract DA 36-039 SC 89099. DA Proj. 3A 99-27-005. UNCLASSIFIED report.

A set of equations describing kinematical relationships among cloud, precipitation, and the wind field are derived in detail. Steady-state profiles of cloud in a non-precipitating tropical atmosphere are presented for 10°C steps of the condensation level; the unresponsiveness of cloud content to vertical air displacements at high altitudes and low temperatures is an explanation for the relative persistence of cirrus clouds. Exact solutions are presented for a cloud and precipitation model that is simplified to include only the problem of precipitation initiation. This model shows that where two processes for precipitation formation exist, the onset of precipitation is controlled only within certain limits by alteration of one of the processes.

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